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Yong-Rak Kim [a], Minki Hong [b], David H. Allen [c] & Seong-Wan Park [d]

[a] Department of Civil Engineering, Kyung Hee University, Yongin-si, Gyeonggi-do, 446-701, South Korea

[b] Department of Civil Engineering, University of Nebraska, Lincoln, NE, 68588-0531, USA

[c] College of Engineering and Computer Science, University of Texas-Pan American, Edinburg, TX, 78539, USA

[d] Department of Civil and Environmental Engineering, Dankook University, Yongin-si, Gyeonggi-do, 448-701, South Korea
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Statistical and dimensional analysis of hot-mix asphalt mixture characteristics on asphalt pavement analyser rutting behaviour

Yong-Rak Kim[a1], Minki Hong[b2], David H. Allen[c3] and Seong-Wan Park[d]*

[a]*Department of Civil Engineering, Kyung Hee University, Yongin-si, Gyeonggi-do 446-701, South Korea;* [b]*Department of Civil Engineering, University of Nebraska, Lincoln, NE 68588-0531, USA;* [c]*College of Engineering and Computer Science, University of Texas-Pan American, Edinburg, TX 78539, USA;* [d]*Department of Civil and Environmental Engineering, Dankook University, Yongin-si, Gyeonggi-do 448-701, South Korea*

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This study presents a statistical and dimensional approach as a simple and efficient means to characterise the effects of individual mixture variables on the performance behaviour resulting from the laboratory tests of hot-mix asphalt (HMA) mixtures. The approach is to find the material and/or mixture variables that significantly affect the performance test results and to further assess the level of significance of variables, with the goal of improving the current understanding of HMA performance tests and their practical applications. To demonstrate the approach, two sets of asphalt pavement analyser (APA) test data from Nebraska and one set of data from Kentucky are statistically analysed based on the multiple linear regression technique. A dimensional analysis based on the Buckingham π-theorem is also performed, and the analysis results are compared with the statistical analysis results. Both analyses present comparable results in that the binder stiffness of HMA mixtures is the most significant variable affecting APA performance test results when the mixtures meet volumetric requirements.

Keywords: hot-mix asphalt; multiple linear regression; dimensional analysis; Buckingham π-theorem; asphalt pavement analyser

Introduction

Performance testing of hot-mix asphalt (HMA) mixtures has been considered a core effort for better design of pavement structures, since it provides more accurate characterisation of properties and performance potential of the paving mixtures. The initial development of the Superpave mixture design under the Strategic Highway Research Program in the early 1990s included various mixture performance tests in the form of Superpave performance testing programme. Before the Superpave programme, the traditional Marshall and Hveem mixture design methods had also required mixture stability tests to provide some measure of the mix quality, even though the Marshall and Hveem stability tests were empirical.

Numerous research projects have demonstrated that the Superpave volumetric mixture design method alone is not sufficient to ensure reliable mixture performance over a wide range of materials, traffic and climatic conditions. In addition, state department of transportation (DOT) engineers and industry practitioners have sought certain form of simple performance tests to help ensure that a quality product is produced. To that end, numerous efforts have been made by a number of researchers through various studies (Witczak *et al.* 2002, Kandhal and Cooley 2003, Christensen and Bonaquist 2004, AASHTO TP62 2007) to develop performance tests of HMA mixtures so that one can better assess mixture quality and conduct pavement structural design in a more optimised manner. Consequently, the recent development of the asphalt mixture performance tester and the mechanistic-empirical pavement design guide programmes include some of the performance tests that account for primary HMA distresses such as fatigue cracking, rutting and thermal cracking.

Performance tests of HMA mixtures generally present a high testing variability. This is because of the significant heterogeneity of the mixtures where multiple phases are mixed in a wide length scale. The high testing variability consequently needs many replicate specimens to draw reasonable conclusions, which is time-consuming and costly. In addition, it has also been recognised that performance test results are sensitive to and controlled by properties of individual mix constituents, interactions between constituents and proportioning of constituents. Therefore, it is attractive to have certain approaches that can provide the mechanical or physical relationships between characteristics of mixture variables and mixture performance behaviour so that the performance testing can be understood and conducted in a more efficient way, which can significantly reduce testing specimens.

*Corresponding author. Email: spark@dankook.ac.kr

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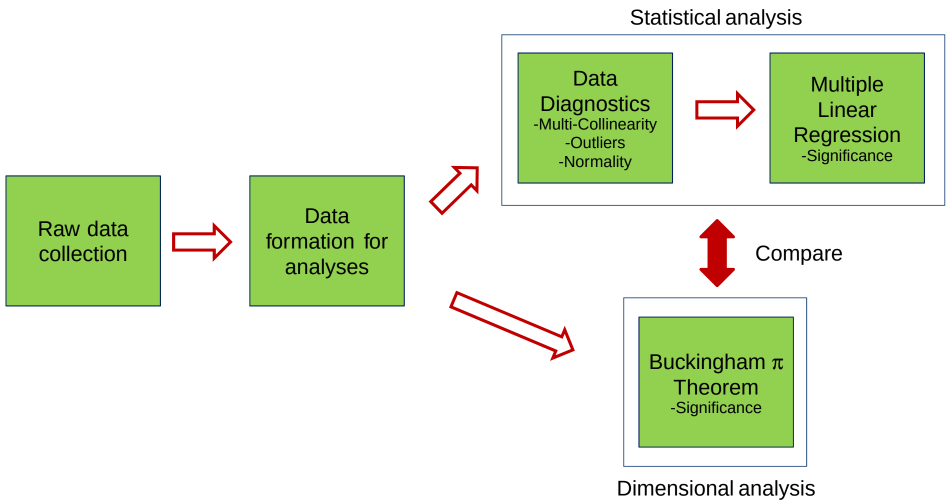

Figure 1. Process of the analysis methods employed for this study.

Recently, some studies have been pursued to develop such useful techniques relating mixture constituents and their composition to engineering properties and performance behaviour of mixtures. For example, various microstructure approaches (Buttlar and You 2001, Masad *et al.* 2001, Guddati *et al.* 2002, Papagiannakis *et al.* 2002, Sadd *et al.* 2003, Soares *et al.* 2003, Dai *et al.* 2005, Kim and Buttler 2005, Kim *et al.* 2006) with the aid of numerical techniques have been actively attempted by researchers. This is because the microstructure approaches can account for the different properties of mixture constituents, the heterogeneity of the mixtures, and the damage evolution characteristics in a more realistic length scale without requiring a large number of laboratory tests of mixture specimens. Microstructure approaches typically rely on individual mixture constituents and microstructure geometry of the mixtures to characterise overall mixture properties. However, the intensive computational efforts, theoretical complexities and somewhat incomplete predicting power of the approaches have also been reported as challenges for their practical implementation.

Alternatively, dimensional analysis seems attractive as a simple means of obtaining relationships that describe some complex phenomenon even without understanding the detail mechanism of the phenomenon. Dimensional analysis can contribute to the construction of a physical model and offers an idea for the form of solutions for theoretical analyses and experimental design. This process thus can potentially find the sensitivity and/or significance among the variables (such as mixture characteristics and performance results) affecting a physical problem (such as HMA performance tests). Similarly, statistical analyses (Kandhal and Mallick 2001, Kandhal and Cooley 2002, McCann and Sebaaly 2003, Tarefder *et al.* 2003, Bausano and Williams 2009, Li *et al.* 2010) including the multiple linear regression analysis have been used when experimental data have several independent variables, as observed in typical HMA performance testing. The general purpose of multiple linear regression analysis is to investigate the linear relationship between several independent variables (i.e. materials and mixture variables associated with performance test) and a dependent variable (i.e. performance test result). Multiple linear regression analysis can also be used to identify the independent variables significantly affecting the performance results by comparing two test statistics: ***F***-ratio and ***t***-ratio, to a specified significance level (Neter *et al.* 1996, Draper and Smith 1998).

This study attempts the statistical and dimensional approach as simple and efficient methods to characterise the effects of individual mixture variables on the overall HMA performance behaviour. The integrated approach presented herein can find the material and/or mixture variables that significantly affect the performance test results and the level of significance of variables without directly relying on the sophisticated but technically challenging mechanistic modelling approaches such as the microstructure modelling. Therefore, with the aid of research outcomes, the current HMA performance tests and their practical applications can be improved based on the better understanding of the physical relationships between mixture variables and performance behaviour resulting from laboratory tests of HMA mixtures.

Research methodology

Figure 1 briefly illustrates the process of the research method employed for this study. A statistical method based on multiple linear regression analysis and a dimensional analysis based on the Buckingham π-theorem

Table 1. APA data-sets collected, their mixture characteristics with the required properties and some basic statistics.

Mixture variables	Nebraska *SP*-4	Nebraska *SP*-5	Kentucky mix
NMAS (mm)	9.5, 12.5	9.5, 12.5	9.5, 12.5
PG binders	64-22, 64-28, 70-28	64-22, 64-28, 70-22, 70-28	64-22, 70-22, 76-22
Air voids (%)			
Maximum, minimum	6.0, 3.5	5.6, 3.7	4.3, 3.8
Mean, standard deviation	4.30, 0.54	4.35, 0.56	4.07, 0.12
Required	3–5	3–5	3–5
Asphalt content (%)			
Maximum, minimum	6.8, 4.6	6.5, 4.5	6.3, 5.4
Mean, standard deviation	5.5, 0.42	5.6, 0.51	5.8, 0.30
Required	N/A	N/A	N/A
CAA1 (%)			
Maximum, minimum	99, 85	100, 91	100, 100
Mean, standard deviation	93, 3.8	97, 1.8	100, 0.0
Required	85	95	100
CAA2 (%)			
Maximum, minimum	98, 80	100, 90	100, 100
Mean, standard deviation	88, 4.9	94, 2.7	100, 0.0
Required	80	90	100
FAA (%)			
Maximum, minimum	50, 45	48, 45	48, 45
Mean, standard deviation	45.4, 0.64	45.5, 0.62	45.9, 1.02
Required	45	45	45
% Passing at the PCS			
Maximum, minimum	73.9, 27.5	62.9, 34.3	57.0, 28.0
Mean, standard deviation	49.8, 8.48	52.7, 7.17	42.4, 8.76
Required	N/A	N/A	N/A
Traffic volume (10^6 ESALs)	1 to <10	≥ 10	≥ 30
No. of APA data collected	91	22	16

were selected for this study. The multiple linear regression analysis has often been used and widely applied to various types of HMA test results, while the Buckingham π-theorem for the dimensional analysis has not frequently been used for the experimental design and analysis of HMA test results despite its simple and straightforward characteristics.

As illustrated in the figure, the two analysis methods were individually applied to the common set of test data. This is to conduct a sensitivity analysis for the variables related to an arbitrary HMA performance test through the application of the dimensional analysis with the same performance test data used for the statistical analysis. Analysis results from the Buckingham π-theorem can then be compared with the results from the multiple linear regression statistical analysis.

To demonstrate the approach presented herein, two sets (i.e. mixture types) of asphalt pavement analyser (APA) data from Nebraska and one set of data obtained from Kentucky were used. The APA test and its results were selected for this study as a representative example of HMA performance tests, since the APA has been widely used in many states to evaluate rutting potential of HMA mixtures. APA test data are relatively abundant than other performance test results, since many state DOTs including Nebraska have performed the APA test for their project mixtures as a quality control/quality assurance tool, because APA testing is very simple, rapid and easy to perform.

Data collection

As mentioned above, two sets of data from Nebraska APA mixtures (designated as *SP*-4 and *SP*-5 hereafter) and one set of APA data from Kentucky were collected from field projects for the analyses. Table 1 presents the data-sets collected, their mixture characteristics with the required material properties and some basic statistics (i.e. maximum, minimum, mean and standard deviation) of key variables considered in this study. As can be seen in the table, Nebraska *SP*-4 was used for roadways with intermediate traffic volume [1–10 million equivalent single axle loads (ESALs)], whereas Nebraska's *SP*-5 and the Kentucky mixture were used for roadways with high-traffic volume.

From the data-set, it was observed that two nominal maximum aggregate sizes (NMAS), 9.5 and 12.5 mm were used for all three mixture types. Three binder performance grades (PG 64-22, 64-28 and 70-28) have been used for Nebraska *SP*-4, while four different (PG 64-22, 64-28, 70-22 and 70-28) and three different (PG 64-22, 70-22 and 76-22) types of PG binders were used to produce Nebraska

SP-5 and the Kentucky mixture, respectively. Table 1 also presents the number of APA data collected for each mixture.

Data formation for analyses

The materials and/or mixture variables to be considered for analysis needed to be selected from the collected raw data and the corresponding materials/mixture information. Based on the results from the dimensional analysis, which is discussed in later sections, six factors (binder PG, aggregate gradation, NMAS, aggregate angularity, air voids and asphalt content) were selected for the statistical analysis. The binder PG represents the binder's stiffness characteristics, and is expected to affect the APA rut data. The size and shape factors of aggregates, such as gradation, angularity and NMAS were included in the analysis. For the mixtures, two variables (air voids and asphalt content) were selected as primary factors because they are crucial indicators identifying a mixture's volumetric characteristics and are also expected to affect the APA rut depth.

For a more detailed statistical analysis, the aggregate angularity factor was categorised into three variables: coarse aggregate angularity value with one or more fractured faces (denoted by CAA1), coarse aggregate angularity value with two or more fractured faces (denoted by CAA2) and fine aggregate angularity (FAA). In the case of aggregate gradation, the gradation factor needed to be quantified in numbers to be implemented in the statistical analyses. In an attempt to quantify the characteristics of aggregate gradation, the percentage of aggregates that passed through the primary control sieve (PCS) in the Bailey method was used. Traditionally, the designation between fine and coarse aggregates is whether or not a particle passed the 4.75-mm sieve (No. 4). In the Bailey method, the coarse and fine designation depends on the NMAS of the mixture. The PCS in the Bailey method is simply obtained by multiplying the NMAS by a factor 0.22. Correspondingly, the PCS in the Bailey method will vary for different types of mixtures by physically

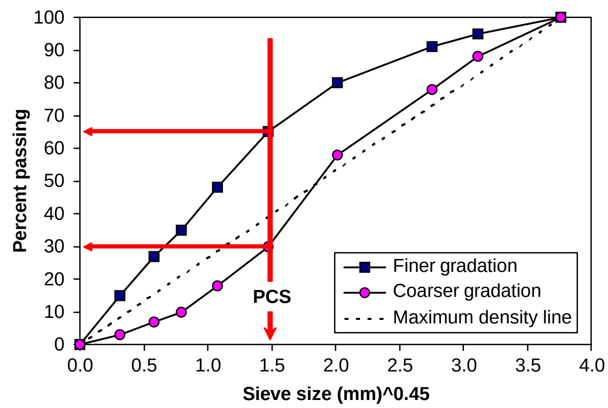

Figure 2. Illustration of the PCS for 12.5-mm NMAS gradations (coarse and fine).

determining the boundary between coarse and fine aggregates in the combined blend (Vavrik *et al.* 2002). Once the PCS has been determined for each mixture, the corresponding percent passing through the PCS is then captured as an indicator to characterise coarseness of the mixture gradation. As exemplified in Figure 2, a finer gradation shows a higher percentage of aggregate passage through the PCS than a coarser gradation.

After all the independent variables were selected, individual data sheets for each mixture (the two Nebraska mixtures and the Kentucky mixture) were developed in a tabular form as shown in Table 2, which shows the *SP*-4 data sheet for the purpose of illustration. Table 2 presents specific values for the independent variables, with the APA rut result as the dependent variable. In the case of the binder PG, the numbers 1, 2 and 3 were used to represent 64-22, 64-28 and 70-28, respectively, for the purpose of statistical analyses. For the other variables, real experimental values obtained were used. Instead of using the APA rut depth (in mm), a different quantity, rut ratio (δ) was used as simply represented by the following equation:

$$\delta = \frac{\mathrm{RD}_T}{N} \times 100, \tag{1}$$

Table 2. Data sheet (*SP*-4) developed for statistical analysis.

No.	NMAS	PG	Air voids (%)	Binder (%)	CAA1	CAA2	FAA	% at PCS	Rut ratio
1	12.5	2	4.2	5.9	94	90	45.7	47.7	0.0156
2	12.5	1	3.9	5.1	89	86	45.4	43.7	0.0222
3	12.5	2	4.8	5.7	99	94	50.1	48.6	0.0251
4	9.5	2	4.0	6.8	95	90	46.9	56.0	0.0280
• • •									
90	12.5	1	5.0	5.6	91	82	45.5	44.9	1.2956
91	12.5	2	4	5.1	95	89	45.2	39.7	1.8115

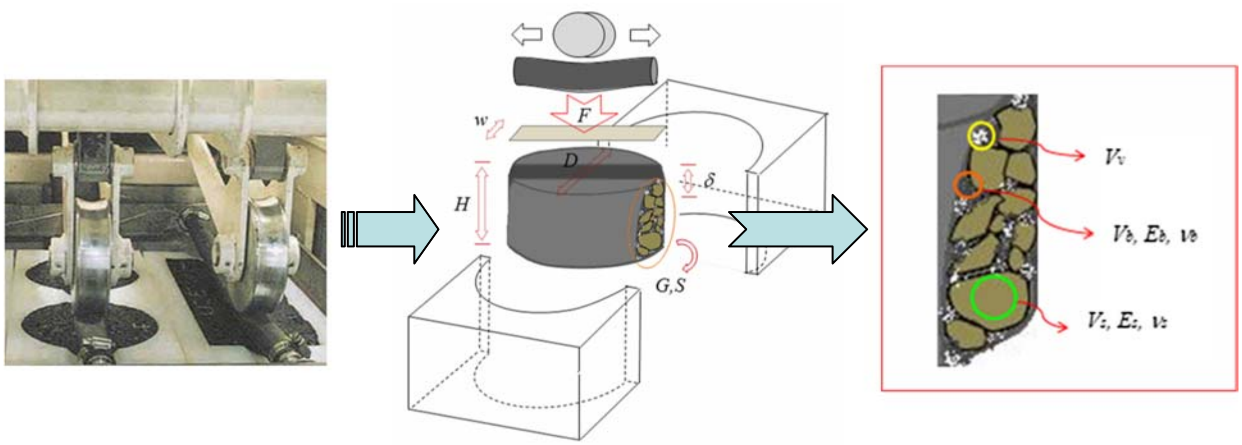

Figure 3. APA testing configuration for the dimensional analysis.

where RD_T = total rut depth (in mm) monitored and N = corresponding number of loading cycles at the RD_T.

The rut ratio is a replacement for the rut depth in this study, because the APA test automatically stopped when the wheel loading reached 8000 cycles before a 12 mm rut depth or when the total rut depth exceeded 12 mm before the 8000-cycle wheel loading. To provide an identical measure of a mixture's rut potential for any case, the rut ratio (δ) was calculated and used.

For the dimensional analysis using the Buckingham π-theorem, a total of nine material-mixture factors related to the APA testing were considered. The HMA specimen for the APA testing was assumed to be an isotropic, elastic composite composed of aggregates, asphalt binder and air voids. Four factors representing the properties of each mixture component [i.e. elastic stiffness of the aggregate (E_s), elastic stiffness of the asphalt binder (E_b), Poisson's ratio of the aggregate (ν_s) and Poisson's ratio of the asphalt binder (ν_b)] and five factors related to the geometry of each mixture component [i.e. volume fraction of air voids (V_v), volume fraction of asphalt binder (V_b), volume fraction of aggregate (V_s), NMAS represented by S and a factor G representing the aggregate gradation] were included for the analysis, as illustrated in Figure 3. Other factors associated with loading [applied force (F) and loading width (w)] and the bulk APA specimen geometry [height (H) and diameter (D) of APA specimen] were also involved in the dimensional analysis, but they were fixed variables from testing performed. The APA rut result (δ), which is the output from the APA testing, was related to various input variables; material properties and the geometric characteristics of each mixture component in a mixture.

To simplify the application of the Buckingham π-theorem, the aggregate angularity factors were excluded, and the same indicator (the percentage of aggregates that passed through the PCS) that was employed for the statistical analysis was used to represent the characteristics of the aggregate gradation by using the factor G. In addition, mechanical behaviour of the asphalt binder was assumed as elastic, although its actual constitutive behaviour is viscoelastic and/or viscoplastic. This assumption was made in this study because the dimensional analysis becomes extremely complicated when the time-dependent material behaviour is involved. It is considered a reasonable assumption for the purpose of this study, since the elastic modulus of binder can still represent the effect of binder stiffness on APA mixture performance.

With all of the variables involved, a data sheet for Nebraska mixture *SP*-4 was developed as shown in Table 3 and used for the analysis in order to find the significance of

Table 3. Data sheet (*SP*-4) developed for dimensional analysis.

No.	Fixed variables	S (mm)	G	E_b (Pa)	V_v (mm^3)	V_s (mm^3)	V_b (mm^3)	δ
1	F (N) = 445	12.5	47.7	3076	55.665	1117.28	152.416	0.0156
2	w (mm) = 25	12.5	43.7	3218	51.689	1137.16	136.512	0.0222
3	D (mm) = 150	12.5	48.6	3359	63.617	1129.21	132.536	0.0251
4	H (mm) = 75	9.5	56.0	3076	53.014	1114.63	157.718	0.0280
•	ν_s = 0.15				•			
•	ν_b = 0.45				•			
•	E_s (GPa) = 60.9				•			
90		12.5	44.9	2401	66.268	1115.95	143.139	1.2956
91		12.5	39.7	2401	53.014	1131.86	140.488	1.8115

Table 4. A typical ANOVA table from the multiple linear regression analysis.

Source	Degree of freedom (DF)	Sum of squares (SS)	Mean square (MS)	***F***-ratio
Regression model	p	SSR	MSR = SSR/p	MSR/MSE
Error	$n - p - 1$	SSE	MSE = SSE/$(n - p - 1)$	
Total	$n - 1$	SSTO		

Note: SSR = regression SS; SSE = error SS; SSTO = total SS; MSR = MS due to regression and MSE = MS due to error.

each input variable affecting the APA test output (i.e. rut ratio) by relating each input to the output. As presented in the table, specific values for each factor of individual APA specimen were obtained. In the case of the elastic stiffness of the asphalt binder, the dynamic modulus at 60°C was obtained by using a dynamic shear rheometer (DSR) in torsional loading mode. Elastic modulus of aggregates was measured by the nano-indentation technique (Khanna *et al.* 2003). Eight measurements were made resulting in a mean of 60.9 GPa and a standard deviation of 4.0 GPa. Since the effect of the elastic stiffness of the aggregates on the APA test results was expected to be trivial compared to the effect of the asphalt binder stiffness due to a considerably large difference in the stiffness of the two materials, the elastic stiffness of aggregate was fixed for this study with the same value of 60.9 GPa. Constant values of 0.15 and 0.45 were assumed and used as Poisson's ratios for the aggregate and binder, respectively.

Statistical analysis by multiple linear regression

Before using the data sheets to perform multiple linear regression for the statistical analysis, three key data diagnostic checks (i.e. multi-collinearity, outliers and normality) were performed. Correlations among independent variables need to be checked before beginning multiple linear regression analysis, because high correlations among variables cause multi-collinearity, which typically leads to a faulty result. Variables demonstrating high correlations were found and modified into another variable so that multi-collinearity could be avoided. The next step was to detect outliers in the data-set. Outliers are extreme observations in comparison to the rest of the data. When there are outliers in a data-set, a statistical analysis is performed with values not representing the overall data. Moreover, outliers affect data normality. Multiple linear regression analysis is valid for a data-set where normality holds. Thus, the investigation of outliers is an important step. To detect outliers, the DFBETAS technique (Heiberger and Holland 2004, Montgomery *et al.* 2006) was employed in this study. After removing the extreme observations (outliers), a data normality check was then performed. When the number of data is less than 5000, the Shapiro–Wilk test (Shapiro and Wilk 1965) is often used for the data normality check. Thus, the Shapiro–Wilk test was utilised in this study to verify data normality.

After completing the data diagnostic tests, the multiple linear regression analysis was conducted. As mentioned earlier, multiple linear regression analysis is a method that finds the statistical model that defines the experimental data based on several independent variables and a dependent variable. The regression analysis outcomes are first processed using the analysis of variance (ANOVA), which yields a test statistic (typically, the ***F***-ratio) that determines whether or not the independent variables explain some of the variation in the dependent variable. A typical format and entities in an ANOVA table from the multiple linear regression analysis with n number of data and p number of independent variables in the model are presented in Table 4. The mean square due to error (MSE) and the mean square due to regression (MSR) are given in the fourth column of the ANOVA table. The ***F***-ratio in the fifth column is simply calculated by dividing MSR by MSE and provides a statistic for testing whether the independent variables explain some of the variation in the response variable (dependent variable).

The significance of the test results is justified by comparing the ***F***-ratio computed from the experimental data with the pre-determined ***F***-ratio based on the significance level (referred to as α value) specified, the number of data collected and the number of independent variables involved. The significance level (α) is specified by users and is typically equal to 0.001, 0.01, 0.05 or 0.10. If the probability, where the ***F***-ratio obtained from the experimental data is equal to or greater than the pre-determined ***F***-ratio, is less than the specified significance level (α value), there is sufficient evidence to say that at least one independent variable contributes to the variation in the dependent variable. Thus, the model resulting from the multiple linear regression analysis is considered to be one where a meaningful relationship exists between a dependent variable and independent variables.

When the testing analysis is significant, a multiple linear regression model relating variables can be formed with parameter estimates of individual significant independent variables. The level of significance of each independent variable is then identified based on a statistic, the ***t***-ratio. The ***t***-ratio is a ratio of a parameter estimate to its standard error. Similar to the ***F***-ratio, the ***t***-ratio is used to assess the significance of individual regression coefficients (parameters). Each computed ***t***-ratio is compared with the pre-determined ***t***-ratio based on the significance level (α value) specified. The probability where the calculated ***t***-ratio is equal to or greater than the pre-determined ***t***-ratio is obtained and compared with the specified significance level. If the probability is less than the significance level, the corresponding independent variable has a significant impact on the dependent variable. Therefore, significant independent variables contributing to the variation in the APA rut results can be found.

Dimensional analysis by Buckingham π-theorem

The Buckingham π-theorem is a key theorem in dimensional analysis. This theorem offers an idea for the form of solutions for theoretical analyses and experimental design. The Buckingham (1914) π-theorem states that 'If there are n variables in a problem and these variables contain m primary physical dimensions (for example, M (mass), L (length), T (time), Θ (temperature)), the equation relating all the variables will have $(n-m)$ dimensionless groups.' The dimensionless groups are typically represented as π groups and they can further be related in a more specific form when actual experimental data (or observations) are available.

For the theorem, there are two conditions: (1) each of the fundamental dimensions must appear in at least one of the n variables and (2) the dimensionless π groups must be independent of each other and no one group should be formed by multiplying together powers of other groups. Using the basic concept of the theorem, a routine procedure can be written as below:

(1) Clearly define the problem and identify important variables to be considered.
(2) Express each of the n variables in terms of its fundamental dimensions $\{M, L, T, \Theta\}$.
(3) Determine the number of π groups, $j = n - m$.
(4) Form j dimensionless π groups and check whether they are all indeed dimensionless.
(5) Express the result in the form of $\pi_1 = \Phi(\pi_2, \ldots, \pi_{n-m})$.
(6) And compare the relationship with the experimental data available.

As presented above, the Buckingham π-theorem is used to show a physical relationship among variables. The dimensionless π-groups are usually found from the m variables according to an intuitive process. Once the equation expressing the relationship between the variables is defined based on the experimental data available, this process can potentially find the sensitivity and/or significance among the variables affecting a physical problem.

Analysis results and discussion

To conduct multiple linear regression analysis, the Statistical Analysis Software was used. Two sets (i.e. *SP*-4 and *SP*-5 mixtures) of APA data from Nebraska and one set of APA data obtained from Kentucky were used to conduct the statistical analysis, while the dimensional analysis was only applied to the *SP*-4 Nebraska data-set. This is because other data-sets were incomplete to accomplish the dimensional analysis. The analysis results from the statistical approach were then compared with the results from the dimensional analysis.

Statistical analysis results of SP-*4 mixture data*

As mentioned earlier and illustrated in Figure 1, data diagnostic tests were performed first. Table 5 presents the correlations among eight independent variables selected for the analysis. Assuming that variables are highly correlated if their correlation factor is greater than 0.5, CAA1 is highly correlated with CAA2, as given in the table. In order to avoid multi-collinearity due to high correlations among variables, variable modification is required. One of the typical ways to modify the variables is to combine two variables into a single variable by addition. Thus, CAA1 was added to CAA2 to produce a single variable, CAA, for this study.

After the multi-collinearity diagnostic test was completed, outliers in the data-set were detected. As mentioned earlier, the DFBETAS technique was employed to search for influential observations (outliers). A total of seven outliers were found and removed from the data-set. The normality of the data-set was then checked through

Table 5. Correlations among independent variables (*SP*-4 mixture).

Variable	NMAS	PG	Air void	Binder content	CAA1	CAA2	FAA	Gradation
NMAS	1	−0.064	−0.073	−0.465	0.101	0.125	−0.064	−0.407
PG		1	−0.067	−0.018	0.036	−0.023	0.060	−0.042
Air void			1	0.022	0.192	0.141	0.082	0.114
Binder content				1	−0.117	0.007	0.361	0.437
CAA1					1	0.781	0.135	−0.019
CAA2	*Symmetry*					1	0.081	−0.037
FAA							1	0.079
Gradation								1

the Shapiro–Wilk test to see whether the data-set satisfied a normal distribution to accomplish the multiple linear regression analysis. The Shapiro–Wilk test is one of general normality tests designed to detect all departures from normality. The test rejects the hypothesis of normality when the *P*-value is less than or equal to 0.05. Failing the normality test allows one to state with 95% confidence that the data do not fit the normal distribution. The Shapiro–Wilk test result confirmed a normal distribution of the data.

Multiple linear regression analysis was then performed using the data-set (a total of 84 APA observations after removing the seven outliers). Table 6 presents the overall significance of the test results, as justified by the ***F***-ratio and *P*-value (Pr. $> F$) obtained from the ANOVA table. By comparing the *P*-value with a given α value, it was possible to determine whether there was at least one independent variable that affected the variation of the dependent variable (i.e. APA rut results). Table 6 also presents the significance of each individual regression coefficient and its parameter estimate by providing ***t***-test results, which were useful to assess the significance of each independent variable in the model. As mentioned previously, if the *P*-value (i.e. Pr. $> |t|$) is less than the specified significance level (α value), the independent variable being considered is a significant factor affecting the APA rut results. Popular levels of significance are 10% (0.1), 5% (0.05), 1% (0.01) and 0.1% (0.001). The lower the significance level, the stronger the evidence required. Choosing level of significance is an arbitrary task, but for many applications, a level of 5% or 1% is usually chosen, for no better reason than that it is conventional. In this study, 1% significance level (i.e. 0.01 of α value) was chosen, since it is one of the two conventional choices providing statistical significance with the stronger evidence. To maintain the consistency of the analysis, the same value of α (0.01) was applied to all cases.

For *SP*-4 mixture, the test statistics showed that a meaningful relationship existed between the APA rut results and a combination of five variables (i.e. NMAS, Binder PG, Air Void, FAA and Gradation) via a multiple linear regression model, since the *P*-value (0.0001) was less than the specified α value (0.01). It should be noted that CAA was excluded in the table, because it was insignificant in the process when forming the multiple linear regression model. Among the variables (excluding the intercept) considered, three variables (NMAS, binder PG and FAA) were found to be significant at the α value of 0.01. By analysing the parameter estimates of the three significant variables, it can be noted that the negative sign of coefficients indicates the reduced rutting susceptibility of mixtures with stiffer binder, larger NMAS and greater FAA value of aggregates in the mixture, which is in agreement with common observations made by many researchers.

Statistical analysis results of SP*-5 mixture data*

For the *SP*-5 mixture, data diagnostic tests revealed a high correlation between CAA1 and CAA2 (0.719) and three outliers. Similar to the *SP*-4 case, CAA1 was added to the CAA2 to produce a single variable (CAA), and all outliers were removed from the data-set. A normality check by the Shapiro–Wilk test was also performed and a normal distribution was confirmed. Table 7 summarises the analysis results including the ANOVA and the significance of individual variables with their parameter estimates. The *P*-value (Pr. $> F$) was 0.003, which was smaller than the specified significance level ($\alpha = 0.01$). Therefore, the model could be formed with a meaningful relationship between the APA rut depth and independent variables. Among the independent variables included in the model, only the binder PG showed significance towards APA rutting.

Statistical analysis results of Kentucky mixture data

As shown earlier in Table 1, the values of CAA1 and CAA2 in the Kentucky mixture were identical (100/100).

Table 6. Analysis results of the *SP*-4 mixture.

	Nebraska data (*SP*-4)				
ANOVA	***F***-ratio = 8.63 Pr. $> F$ = 0.0001				
	Unstandardised coefficients		Standardised coefficients		
Variables	Coefficient	Standard error	Coefficient	***t***-Ratio	Pr. $> \|t\|$
Intercept	8.156	2.167	0	3.76	0.0003
NMAS	−0.062	0.014	−0.465	−4.63	0.0001
Binder PG	−0.129	0.036	−0.333	−3.63	0.0005
Air void	−0.056	0.036	−0.144	−1.57	0.1205
FAA	−0.142	0.047	−0.283	−3.02	0.0035
Gradation	−0.006	0.003	−0.248	−2.47	0.0158

Table 7. Analysis results of the *SP*-5 mixture.

	Nebraska data (*SP*-5)				
ANOVA	***F***-ratio = 7.49 Pr. > ***F*** = 0.003				
	Unstandardised coefficients		Standardised coefficients		
Variables	Coefficient	Standard error	Coefficient	***t***-Ratio	Pr. > |***t***|
Intercept	0.505	0.314	0	1.61	0.128
Binder PG	−0.136	0.033	−0.804	−4.17	0.001
% Binder	−0.092	0.057	−0.290	−1.62	0.126
Gradation	0.011	0.005	0.470	2.36	0.032

Therefore, the two angularity values were replaced with one variable, CAA to conduct statistical analyses of the Kentucky APA data. Due to this fact, data diagnostic tests for the Kentucky mixture revealed no high correlation among variables. Two outliers were removed, and data normality was confirmed. Table 8 presents the multiple linear regression analysis results.

Test statistics presented in Table 8 indicated that a meaningful relationship exists between the APA rut results and the independent variables, since the P-value (0.001) was clearly less than the α value (0.01). Among the independent variables involved, only one variable (binder PG) was significant (Pr. > |$\boldsymbol{t}$|: 0.001). The binder PG produced a negative effect on performance, as observed in the Nebraska mixtures. All three analyses results given in Tables 6–8 obviously demonstrate that the APA rutting performance is sensitive to the binder PG. Stiffer binders (higher PG grade) were less susceptible to APA rutting than softer ones, as expected.

Dimensional analysis results of SP*-4 mixture data*

The HMA specimen for the APA testing was modelled as an elastic composite made up of aggregates, asphalt binder and air voids, as illustrated in Figure 3. For the dimensional analysis based on the Buckingham π-theorem, the first step was to identify a functional relationship for the APA rut depth in terms of the variables that influence it. All variables included are presented in Table 9. As presented in the table, the APA rutting performance (rut ratio δ) is related to input variables categorised into three groups: variables associated with load (F), geometry (w, D, H, G, S, V_v, V_s and V_b) and material properties (ν_s, ν_b, E_s and E_b).

In order to simplify the analysis, the HMA specimen was assumed to be an isotropic, linear elastic and three-phase (air, binder and aggregate) material subjected to a static force (F). Table 9 also presents the characteristics of each input variable: fixed or varied for the analysis. The variable characteristics were determined by an examination of the APA test data (i.e. data obtained from *SP*-4 mixtures) used for this analysis. The APA testing was performed by fixing the applied load (F), geometry (w, D and H) and material properties (ν_s, ν_b and E_s). All the other variables (i.e. G, S, V_v, V_s, V_b and E_b) were different for each APA specimen and consequently were regarded as variables investigated to monitor their significance to the APA rut results (δ).

As noted above, a total of 14 variables (independent and dependent) were incorporated, six of which were considered potentially significant testing input variables affecting the APA rut results. The units for all 14 variables are presented in Table 9. As given in the table, there are only three independent physical units: $[L]$, $[M]$ and $[T]$.

Table 8. Analysis results of the Kentucky mixture.

	Kentucky data				
ANOVA	***F***-ratio = 46.13 Pr. > ***F*** = 0.001				
	Unstandardised coefficients		Standardised coefficients		
Variables	Coefficient	Standard error	Coefficient	***t***-Ratio	Pr. > |***t***|
Intercept	0.163	0.053	0	3.09	0.012
NMAS	0.002	0.001	0.185	2.13	0.059
Binder PG	−0.015	0.002	−0.813	−8.59	0.001
FAA	−0.002	0.001	−0.179	−1.98	0.076

Table 9. Summary of variables in APA testing for the dimensional analysis.

Variable	Notation	Category (input/output)	Unit	Fixed or varied	Value if fixed
Applied load	F	Load (input)	N (newton, kg m/s^2) – $[M][L][T^{-2}]$	Fixed	445 N
Width of the rubber hose	w	Geometry (input)	m – $[L]$	Fixed	0.025 m
Diameter of the APA specimen	D	Geometry (input)	m – $[L]$	Fixed	0.150 m
Height of the APA specimen	H	Geometry (input)	m – $[L]$	Fixed	0.075 m
Gradation	G	Geometry (input)	Unitless	Varied	
NMAS	S	Geometry (input)	m – $[L]$	Varied	
Volume of air voids	V_v	Geometry (input)	m^3 – $[L^3]$	Varied	
Volume of aggregate	V_s	Geometry (input)	m^3 – $[L^3]$	Varied	
Volume of binder	V_b	Geometry (input)	m^3 – $[L^3]$	Varied	
Poisson's ratio of aggregate	ν_s	Material property (input)	Unitless	Fixed	0.15
Poisson's ratio of binder	ν_b	Material property (input)	Unitless	Fixed	0.45
Elastic modulus of aggregate	E_s	Material property (input)	Pa (pascal, kg/m s^2) – $[M][L^{-1}][T^{-2}]$	Fixed	60.9×10^9 Pa
Elastic modulus of binder	E_b	Material property (input)	Pa (pascal, kg/m s^2) – $[M][L^{-1}][T^{-2}]$	Varied	
APA rut ratio	δ	Geometry (output)	m – $[L]$	Varied	

Then, as illustrated in the previous sub-section: *Dimensional analysis by Buckingham π-theorem*; a total of 11 ($14 - 3 = 11$) dimensionless parameters denoted as the π-group can be formed, and the resulting model can be expressed as

$$\pi_1 = \Phi(\pi_2, \pi_3, \ldots, \pi_{11}). \tag{2}$$

Each π needs to be a dimensionless parameter, as expressed by

$$\pi_1 = \frac{\delta}{H}, \pi_2 = \frac{F}{E_s wD}, \pi_3 = \frac{F}{E_b wD}, \pi_4 = \frac{V_v}{V_v + V_s + V_b},$$
$$\pi_5 = \frac{V_s}{V_v + V_s + V_b}, \pi_6 = \frac{V_b}{V_v + V_s + V_b}, \pi_7 = \frac{w}{D},$$
$$\pi_8 = \nu_s, \pi_9 = \nu_b, \pi_{10} = G, \pi_{11} = \frac{S}{w}. \tag{3}$$

Therefore, Equation (2) can be rewritten as

$$\frac{\delta}{H} = \Phi\left(\frac{F}{E_s wD}, \frac{F}{E_b wD}, \frac{V_v}{V}, \frac{V_s}{V}, \frac{V_b}{V}, \frac{w}{D}, \nu_s, \nu_b, G, \frac{S}{w}\right), \tag{4}$$

where $V = V_v + V_s + V_b$.

Equation (4) is the final form of the model with one output and 10 input parameters. Among the 10 input parameters, only six parameters ($F/E_b wD$, V_v/V, V_s/V, V_b/V, G and S/w) were varied and considered potentially significant input parameters affecting the APA output parameter (δ/H). In order to find further relationships and the level of significance of each of these six input parameters to the output parameter, *SP*-4 APA experimental data were used to plot the input–output relations as shown in Figure 4. All fitting curves presented in the figure were found with simple power functions. The strength of the relationship between each input and the output was then quantified by the R^2 values, which infer the ranking order of each variable in terms of its significance to the APA rut results. As presented in the figure, the binder stiffness is the factor most related to the APA rut depth, which is in good agreement with the general findings observed from the statistical analysis.

Summary and conclusions

Based on this study, the following summary and conclusions can be drawn:

(1) Two analysis methods (statistical and dimensional) were applied to the same set of HMA performance test data to characterise the effects of individual mixture variables on the performance behaviour and to further assess the level of significance of variables. For this study, two sets of APA data from Nebraska and one set of APA

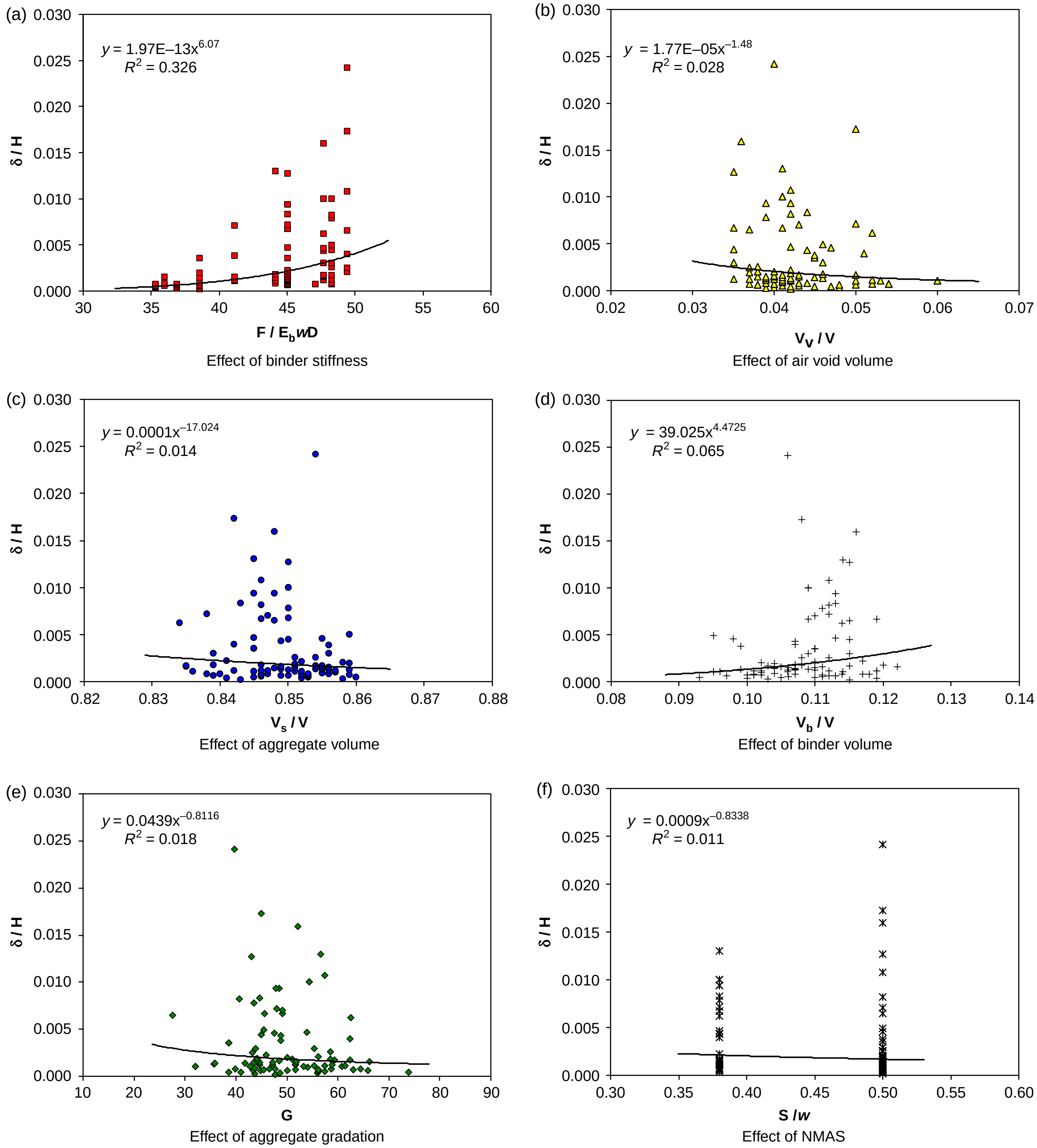

Figure 4. Relationship between δ/H and other individual independent variables.

data obtained from Kentucky were collected and used to implement the two analysis methods.

(2) With the limited data-sets used in this study, both the statistical analysis and the dimensional analysis present comparable results in that the binder stiffness of HMA mixtures is the most significant variable affecting APA rut performance results. The identical analysis results between the two approaches imply the reasonableness of the dimensional analysis method. The Buckingham π-theorem demonstrated its characteristics as a simple means of obtaining relationships that describe some complex phenomenon even without understanding the detailed mechanism of the phenomenon.

(3) Based on the analysis results obtained from this study, it can be implied that APA testing is sensitive to the binder stiffness of a mixture, while the effects of aggregates on HMA rutting performance is not sensitively captured when the mixtures meet volumetric requirements. This tendency is more obvious from high-traffic-volume roadway mixtures where higher quality aggregates are used to meet design requirements. However, it is somewhat premature to make definite conclusions at this stage. Additional data would be necessary to validate findings.

(4) The integrated approach presented in this paper demonstrates its potential applicability to general mixture tests (both fundamental and simulative) in characterising the effects of individual mixture variables on the overall behaviour. It is expected that fundamental test results can provide more informative insights to examine the roles of individual mixture variables and testing conditions on overall mixture characteristics. This study exemplifies only the APA test data for the demonstration purpose; however, the technique can be applied to other tests to identify the material and/or mixture variables that significantly affect test results and to determine the level of significance in a simple manner.

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Notes

1. Email: ykim3@khu.ac.kr
2. Email: mingkee@bigred.unl.edu
3. Email: allendh@utpa.edu

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